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We study on-shell decays of light vector meson resonances ρ , K^* and ϕ in the framework of chiral constituent quark model using resummation calculations. Such studies are necessary for showing that chiral dynamics works well at this energy scale. The effective action is derived by proper vertex method, where resummation of all orders of momentum expansion is accomplished. The loop effects of pseudoscalar meson are also taken into account, which play an important role at this energy scale. The numerical results agree well with the experimental data. A new method to explore the Chiral Symmetry Spontaneously Breaking (CSSB) is proposed. It is found that the unitarity of the effective meson theory resulted from resummation derivations demands an upper-limit to the momentum of vector meson. This upper-limit, being critical point, is just the energy scale of CSSB, and is found to be flavor-dependent.

12.39.-x, 11.30.Rd, 11.30.Qc, 12.40.Yx, 12.40.Vv, 13.25.-k, 13.75.Lb

I. INTRODUCTION

The Chiral Symmetry Spontaneously Breaking (CSSB) is an important feature in the theory of QCD, which governs the dynamics of hadrons at very low energies. It is generally believed that there is an energy scale $\Lambda_{\text{CSSB}} \simeq 2\pi F_\pi \simeq 1.2\text{GeV}$, below which the chiral symmetry $SU(3)_L \times SU(3)_R$ is spontaneously breaking to $SU(3)_{L+R}$ associated with eight Goldstone bosons [1–3]. Under this strong symmetry constraint, effective Lagrangian of mesons, \mathcal{L}_{eff} , can be constructed with numbers of parameters [1,4]. This is the base of the chiral perturbation theory (ChPT) [4], and hence ChPT can be thought of as a rigorous QCD theory. As the typical energy scale p for corresponding hadron dynamics is much less than Λ_{CSSB} , i.e., $p/\Lambda_{\text{CSSB}} \ll 1$, it serves as a good approximation to take a few leading- and next to the leading terms in \mathcal{L}_{eff} in p –(or in momentum, or in derivative) expansion. Thus, the ChPT calculations become to be practicable. Usually, in practice, the calculations in ChPT are up to $O(p^4)$. The predictions of such $O(p^4)$ -calculations describe pseudoscalar meson physics at very low energies quite well.

However, as the energies go up to the meson resonance region (say $E \sim M_\rho$), two novel ingredients should be taken into account: 1, the meson resonances will be excited and they should emerge in the theory; 2, since the energies go up, the p/Λ_{CSSB} will be not much less than 1. For ρ -decays, $p \sim 0.77\text{GeV}$ and $p/\Lambda_{\text{CSSB}} \sim 0.64$; for K^* , $p \sim 0.892\text{GeV}$ and $p/\Lambda_{\text{CSSB}} \sim 0.74$; for ϕ , $p \sim 1.02\text{GeV}$ and $p/\Lambda_{\text{CSSB}} \sim 0.85$. In these cases, the calculations based on taking only a few leading terms in p -expansion (i.e., so called $O(p^4)$ -calculations) will no longer be legitimate. In order to investigate the vector meson decays, all terms of the p -expansions should be taken into account, and then be summed over. We will call it resummation studies on the corresponding vector meson processes. Obviously, for vector meson

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resonance physics at low energies, such resummation studies are necessary and meaningful even though it may be a heavy work. If such studies based on chiral expansion can be performed and the results are reasonable well, then one can conclude that the chiral dynamics (or chiral effective Lagrangian theory) works up to this vector meson energy scale. If not, we will have no reason to think so.

Another motive in this paper is to explore the CSSB of QCD in a non-trivial and realistic QCD-inspired model. CSSB is a prior hypothesis in ChPT. Hence, the success of ChPT provides an indirect evidence for existence of CSSB in QCD. The mechanism of CSSB has been widely discussed in the literature [5] [6]. However, that how to prove it and how to derive and then to determine the critical energy scale Λ_{CSSB} from the fundamental QCD theory still remain to be settled [7]. Therefore, it is still interesting and meaningful to study this subject in more realistic models and in new non-perturbative methods. CSSB in QCD could be thought of as a kind of quantum phase transition phenomenon in the quantum field theories, which is caused by quantum fluctuations in the system [8]. It is well known that as p below Λ_{CSSB} (or after CSSB), the quantum dynamical freedoms are meson fields and the dynamics is described in chiral effective meson Lagrangian. The S -matrices of this Lagrangian field theory have to be unitary, which belongs to the first principle requirement in quantum theories. Thus, as one had a chiral effective meson Lagrangian with all order-terms in p - (or space-time derivative-) expansions, the following question can be asked: In what range of $\sqrt{p^2}$ the S -matrices yielded by the Feynman rules of the theory are unitary? The answer will lead to the determination of Λ_{CSSB} because the upper-limit of this p -range should just be Λ_{CSSB} . In other words, as above this p -upper-limit, i.e., Λ_{CSSB} , the quantum field description of this chiral effective meson Lagrangian system will collapse. This is precisely a critical phenomenon. In conception of Heisenberg's uncertainty principle, the quantum fluctuations of the system in the coordinate space are arisen from its momentum p : larger distance physics associating with relatively smaller quantum fluctuations corresponds to smaller p , and smaller distance one with larger fluctuations corresponds to larger p . Thus, for a quantum field system, it may transfer from order phase to disorder phase along with p -increasing, and then the critical energy scale emerges in the description of the dynamics. It is meaningful and interesting to reveal this scale by applying non-perturbation method to a quantum field system and by examining the unitarity of the theory. In this paper we shall try to realize this idea, i.e., we shall use the resummation derivation method to explore the unitarity of the chiral constitute quark model with vector mesons (see below), and then to determine the critical scale Λ_{CSSB} .

In the literature, there are several schemes to extend the chiral symmetry considerations to be including vector meson resonances, and then the corresponding ChPT-like effective theories with 0^- and 1^\pm mesons can be constructed and studied [9] [10]. Because there are huge number of unknown-parameters in high order terms of p -expansion in this kind of theories, it is impracticable to perform resummation studies in the formalism of ChPT-like theories with vector mesons. Actually, most of all calculations in the literature in these ChPT-like theories are limited to be of $O(p^4)$ or $O(p^6)$ and to be of the leading order of $1/N_c$ -expansion [11]. This situation, of course, is not satisfactory for the studies of vector meson physics even though the theories seem to be model-independent. In this paper we try to provide a phenomenon study based on systematical resummation calculations to processes of $\rho \rightarrow \pi\pi$, $\rho^0 \rightarrow e^+e^-$, $K^* \rightarrow K\pi$, $\phi \rightarrow K^+K^-$ and $\phi \rightarrow K_L^0 K_S^0$ in a realistic QCD-inspired model.

As a QCD-inspired model, the Chiral Quark Model (ChQM) (or Nambu-Jona-Lasinio version models and its extensions) has been extensively studied in hadron physics [3,5,12–19]. The starting point of the model is a chiral constitute quark lagrangian with dynamical Goldstone bosons [3]. The spin-1 mesons are included into the model by using the WCCWZ realization [20,21]. In refs. [17–19], two of us have investigated this model. The follows have been achieved and revealed in refs [17–19]: 1, The parameters in the effective meson lagrangian derived from the ChQM can be fixed by meeting the requirements of KSFR sum rule [22], Zweig rule forbidden to $\phi \rightarrow \pi\pi$, beta decay of neutron and by introducing constitute quark mass parameter as input; 2, The low energy limit of ChQM is consistent with ChPT, and hence ChQM is of a legitimate QCD-inspired model at very low energies; 3, It has been proved that if the constitute quark mass is larger than $m_\rho/2$, the theory at chiral limit is unitary; 4, Since both constitute quarks and the Goldstone bosons are dynamical freedom fields in the ChQM, in the calculations at one loop level, the contributions of the Goldstone boson loops have to be taken into account besides ones from quark loops. Considering the contributions of quark loops and ones of Goldstone boson loops are of $O(N_c)$ and $O(1)$ in $1/N_c$ -expansion respectively, consequently, any consistent loop-expansion calculations in the ChQM must include the contributions from the next to leading order in $1/N_c$ -expansion in the model. This point has been revealed for some processes in the model. 5, Particularly, the resummation calculations of p -expansion and the calculations up to the next to leading order of $1/N_c$ -expansion to vector meson decay processes are formulated. We use the proper vertex expansion method rather than using Schwinger proper time method [23,24] to deal with the quark loop contributions. Thus, we fortunately have a realistic QCD-inspired model in which the resummation calculations can be consistently performed in practice. In this paper we shall recapture the resummation studies on ρ -decays in refs. [17–19] and make it more precise, and then extend it to K^* -, ϕ -decay processes. Furthermore, we shall use large- N_c expansion and optic theorem to explore the unitarity condition of the theory, and use the Feynman rules to obtain the transition

amplitudes of vector meson decays. And then the Λ_{CSSB} is determined. This determination is regularization scheme free.

The contents of this paper are organized as following: In section II we introduce the model with giving the notations; Section III is devoted to illustrate the proper vertex expansion; In section IV, the kinetic terms of vector mesons are derived; Section V, the quark loop contributions to vector meson decays; Section VI, the Goldstone boson loop contributions to vector meson decays; Section VII, the numerical results; Section VIII, unitarity and large N_c expansion. A necessary condition for the unitarity of the meson theory deduced from ChQM is revealed in this section; Section IX, determination of Λ_{CSSB} : i.e., applying the necessary condition of the unitarity, the upper limit of p is derived, and then Λ_{CSSB} is determined. Finally, we provide a brief summary and discussion. In the Appendices, we provide derivations of the low-energy limit of the theory, and show how to perform parametrization of the quadratic divergence emerged in the meson loop calculations in the text. The paper is self-consistent.

II. THE MODEL

For understanding the hadron physics below CSSB scale, Manohar and Georgi provides a QCD-inspired description on the simple constituent quark model [3] (call it as simple-ChQM hereafter). At chiral limit, it is parameterized by the following $SU(3)_V$ invariant chiral constituent quark Lagrangian

$$\mathcal{L}_\chi = i\bar{q}(\not{\partial} + \not{F} + g_A \not{A} \gamma_5)q - m\bar{q}q + \frac{F^2}{16} \langle \nabla_\mu U \nabla^\mu U^\dagger \rangle. \quad (2.1)$$

Here $\langle \cdots \rangle$ denotes trace in $SU(3)$ flavor space, $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ are constituent quark fields, $g_A = 0.75$ is fitted by beta decay of neutron. The Δ_μ and Γ_μ are defined as follows,

$$\begin{aligned} \Delta_\mu &= \frac{1}{2}[\xi^\dagger(\partial_\mu - ir_\mu)\xi - \xi(\partial_\mu - il_\mu)\xi^\dagger], \\ \Gamma_\mu &= \frac{1}{2}[\xi^\dagger(\partial_\mu - ir_\mu)\xi + \xi(\partial_\mu - il_\mu)\xi^\dagger], \end{aligned} \quad (2.2)$$

and covariant derivative are defined as follows

$$\begin{aligned} \nabla_\mu U &= \partial_\mu U - ir_\mu U + iUl_\mu = 2\xi\Delta_\mu\xi, \\ \nabla_\mu U^\dagger &= \partial_\mu U^\dagger - il_\mu U^\dagger + iU^\dagger r_\mu = -2\xi^\dagger\Delta_\mu\xi^\dagger, \end{aligned} \quad (2.3)$$

where $l_\mu = v_\mu + a_\mu$ and $r_\mu = v_\mu - a_\mu$ are linear combinations of external vector field v_μ and axial-vector field a_μ , ξ associates with non-linear realization of spontaneously broken global chiral symmetry $G = SU(3)_L \times SU(3)_R$ introduced by Weinberg [20],

$$\xi(\Phi) \rightarrow g_R \xi(\Phi) h^\dagger(\Phi) = h(\Phi) \xi(\Phi) g_L^\dagger, \quad g_L, g_R \in G, \quad h(\Phi) \in H = SU(3)_V. \quad (2.4)$$

Explicit form of $\xi(\Phi)$ is usually taken as

$$\xi(\Phi) = \exp \{i\lambda^a \Phi^a(x)/2\}, \quad U(\Phi) = \xi^2(\Phi), \quad (2.5)$$

where $\lambda^1, \dots, \lambda^8$ are $SU(3)$ Gell-Mann matrices in flavor space, and the Goldstone bosons Φ^a are treated as pseudoscalar meson octet:

$$\Phi(x) = \lambda^a \Phi^a(x) = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \quad (2.6)$$

The transformation law under $SU(3)_V$ are

$$\psi \longrightarrow h(\Phi)\psi, \quad \Delta_\mu \longrightarrow h(\Phi)\Delta_\mu h^\dagger(\Phi), \quad \Gamma_\mu \longrightarrow h(\Phi)\Gamma_\mu h^\dagger(\Phi) + h(\Phi)\partial_\mu h^\dagger(\Phi). \quad (2.7)$$

Thus the Lagrangian (2.1) is invariant under $G_{\text{global}} \times G_{\text{local}}$. There is only one parameter in this simple model, i.e., constitute quark mass m . With appropriate choice of m -value, the coefficients in ChPT, $L_1, L_2, L_3, L_9, L_{10}$,

have been derived in refs [13,16]. The results shown that the simple-ChQM is consistent with ChPT. Therefore, it is substantial to take the formulation of ChQM as our starting point.

For our purposes, the simple-ChQM must be extended to include lowest vector meson resonances and go beyond the chiral limit. The mass difference of constituent quarks with different flavors is assumed to be caused by current quark masses. The light quark mass matrix $\mathcal{M} = \text{diag}\{m_u, m_d, m_s\}$ is usually included in external spin-0 fields, i.e., $\tilde{\chi} = s + ip$, where $s = s_{\text{ext}} + \mathcal{M}$, s_{ext} and p are scalar and pseudoscalar external fields respectively. The chiral transformation for $\tilde{\chi}$ is $\tilde{\chi} \rightarrow g_R \tilde{\chi} g_L^\dagger$. Thus $\tilde{\chi}$ and $\tilde{\chi}^\dagger$ together with ξ and ξ^\dagger can form $\text{SU}(3)_V$ invariant quantities

$$S = \frac{1}{2}(\xi^\dagger \tilde{\chi} \xi^\dagger + \xi \tilde{\chi}^\dagger \xi), \quad P = \frac{1}{2}(\xi^\dagger \tilde{\chi} \xi^\dagger - \xi \tilde{\chi}^\dagger \xi), \quad (2.8)$$

which are scalar and pseudoscalar respectively. Then the current-quark-mass-dependent term is written

$$-\bar{q}Sq - \kappa \bar{q}P\gamma_5 q, \quad (2.9)$$

which goes back to standard quark mass term of QCD Lagrangian, $-\bar{\psi}\mathcal{M}\psi$ (ψ is the corresponding current quark fields), before CSSB at high energy for arbitrary κ . It means that the symmetry and some underlying constraints of QCD can not fix the couplings between pseudoscalar mesons and constituent quarks. Hence κ is treated as an initial parameter of the model and will be fitted phenomenologically.

From the viewpoint of chiral symmetry only, an alternative scheme for incorporating vector mesons was suggested by Weinberg [20] and developed by Callan, Coleman et al [21]. In this treatment, vector meson resonances V_μ transform homogeneously under $\text{SU}(3)_V$,

$$V_\mu \rightarrow h(\Phi)V_\mu h^\dagger(\Phi), \quad (2.10)$$

where

$$V_\mu(x) = \lambda \cdot \mathbf{V}_\mu + \lambda^0 V_\mu^0 = \sqrt{2} \begin{pmatrix} \frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \phi_\mu \end{pmatrix}, \quad (2.11)$$

and $\lambda^0 = \sqrt{\frac{2}{3}}$. Then the simple-ChQM is extended to a chiral quark model including both pseudoscalar mesons and the lowest meson resonances, which will be called ChQM simply hereafter. ChQM is parameterized by the following $\text{SU}(3)_V$ invariant Lagrangian

$$\mathcal{L}_\chi = i\bar{q}(\not{\partial} + \not{F} + g_A \not{A}\gamma_5 - i\not{V})q - m\bar{q}q - \bar{q}Sq - \kappa \bar{q}P\gamma_5 q + \frac{F^2}{16} \langle \nabla_\mu U \nabla^\mu U^\dagger \rangle + \frac{1}{4}m_0^2 \langle V_\mu V^\mu \rangle. \quad (2.12)$$

We can see that there are five initial parameters g_A , m , κ , F and m_0 in ChQM (F will be renormalized). These parameters can not be determined by symmetry but can only be fitted by experiment.

III. PROPER VERTEX EXPANSION

In chiral quark model, low energy effective action of light hadrons is generated through loop effects of constituent quarks. The usual way to obtain the effective action is in path integral. Integrating out degrees of freedom of quarks, we obtain a determinant and then regularize it by Schwinger proper time method [23] or heat kernel method [24]. In this framework, the effective action is expanded in powers of momentum of mesons, and the calculations to $O(p^4)$ are practicable [15]. However, it is very difficult to calculate more higher order contributions of momentum expansion. Actually, it is impracticable. Instead of it, we shall derive the effective action by computing the loop effects of constituent quarks directly. In this way, the calculations are expansions of loops, or of proper vertices of external fields rather than momentum expansions. We call this method as proper vertex expansion following refs. [18,19], in which all terms in p -expansion are caught for concrete processes, and hence the corresponding calculations are of resummation of p -expansion.

The quark part of Lagrangian (2.12) can be divided into two parts:

$$\begin{aligned} \mathcal{L}_\chi^q &= \mathcal{L}_\chi^0 + \mathcal{L}_\chi^I, \\ \mathcal{L}_\chi^0 &= \bar{q}(i\not{\partial} - m - \mathcal{M})q, \\ \mathcal{L}_\chi^I &= \bar{q}\not{V}q + \bar{q}(i\not{F} - \not{S}')q + \bar{q}(ig_A \not{A} - \kappa \not{P})\gamma_5 q, \end{aligned} \quad (3.1)$$

where $S' = S - \mathcal{M}$.

As a consequence of the free part \mathcal{L}_χ^0 , the propagators of constituent quarks are

$$\overline{q_i^a(x)q_j^b(y)} = \delta_{ab} \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot (x-y)} \frac{i(\not{p} + M_a)_{ij}}{p^2 - M_a^2 + i\epsilon} = \delta_{ab} i S_{Fij}^a(x-y), \quad (3.2)$$

where the flavor index $a, b = 1, 2, 3$ or u, d, s , and $M_a = m + m_a$. That is, the propagators of quarks are flavor-dependent.

The effective action describing meson interaction can be partially obtained via loop effects of constituent quarks

$$\begin{aligned} e^{iS_{\text{eff}}^q} &= \langle 0 | T_q e^{i \int d^4 x \mathcal{L}_\chi^1(x)} | 0 \rangle \\ &= \sum_{n=1}^{\infty} i \int d^4 p_1 \frac{d^4 p_2}{(2\pi)^4} \cdots \frac{d^4 p_n}{(2\pi)^4} \tilde{\Pi}_n(p_1, \dots, p_n) \delta^4(p_1 + p_2 + \cdots + p_n) \\ &\equiv i \Pi_1(0) + \sum_{n=2}^{\infty} i \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_{n-1}}{(2\pi)^4} \Pi_n(p_1, \dots, p_{n-1}), \end{aligned} \quad (3.3)$$

where T_q is time-order product of constituent quark fields, $\tilde{\Pi}_n(p_1, \dots, p_n)$ is one-loop effects of constituent quarks with n external fields, p_1, p_2, \dots, p_n are their four-momentum, and

$$\Pi_n(p_1, \dots, p_{n-1}) = \int d^4 p_n \tilde{\Pi}_n(p_1, \dots, p_n) \delta^4(p_1 + p_2 + \cdots + p_n). \quad (3.4)$$

Getting rid of all disconnected diagrams, we have

$$\begin{aligned} S_{\text{eff}} &= \sum_{n=1}^{\infty} S_n, \\ S_1 &= \Pi_1^c(0), \\ S_2 &= \Pi_2^c(p) + \frac{F^2}{16} \langle \nabla_\mu U \nabla^\mu U^\dagger \rangle + \frac{1}{4} m_0^2 \langle V_\mu V^\mu \rangle \\ S_n &= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 p_{n-1}}{(2\pi)^4} \Pi_n^c(p_1, \dots, p_{n-1}), \end{aligned} \quad (n \geq 3) \quad (3.5)$$

where "c" denotes "connected part", and two non-quark terms in eq.(2.12) have been added to obtain complete effective action. Obviously, in eq. (3.5), the effective action S_{eff} is expanded in powers of number of external vertex and expressed as integral over external momentum. Hereafter we shall call this method proper vertex expansion, and call S_n n -point effective action. In terms of proper vertex expansion, the effective actions include informations from all orders of chiral expansion. That is, we can do resummation of momentum expansion by this method. That is what we need.

For simplicity, we shall use the good approximation $m_u = m_d = 0$, which means the flavor index 1(u) and 2(d) are equivalent for all kind of quantities. In ref. [19] and [18], what is mainly studied is the SU(2) sector of the theory. So, m_s plays no role in such kind of studies. Since $m_u = m_d = 0$, the effective actions can be put into simple forms which are not flavor-related. For SU(3) case, we shall assume $m_s \neq 0$, which, as we can see later, is necessary for a unitary theory when $\phi(1020)$ physics is considered. therefore, the effective actions will be complicated and flavor-related.

The next two sections are calculations of some effective actions. Sect.IV is about kinetic terms of vector mesons, and Sect.V is about tree graphs for vector mesons decays.

IV. KINETIC PART OF VECTOR MESON ACTION

In the effective action, the kinetic part of vector mesons S_{kin} can be derived from the two-point diagram as follow (fig.1). Using Feynman rules generated from eq.(3.1), We find that the kinetic action is

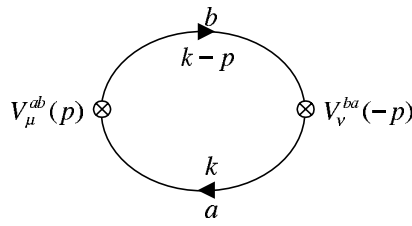


FIG. 1. Two-point diagram of vector mesons.

$$\begin{aligned}
S_{\text{kin}} &= \frac{i}{2} N_c \sum_{ab} \int \frac{d^4 p_1 d^4 p_2}{(2\pi)^4} \delta(p_1 + p_2) V_\mu^{ab}(p_1) V_\nu^{ba}(p_2) T_{ab}^{\mu\nu}(p_2) \\
&= \frac{1}{4} \sum_{ab} \int \frac{d^4 p}{(2\pi)^4} V_\mu^{ab}(p) V_\nu^{ba}(-p) [a_1 g^{\mu\nu} - a_2 g^{\mu\nu} p^2 + b_2 p^\mu p^\nu + a_3(p^2) g^{\mu\nu} p^2 - b_3(p^2) p^\mu p^\nu].
\end{aligned} \tag{4.1}$$

Here V_μ^{ab} refers to the (ab) element of matrix V_μ (2.11), and

$$T_{ab}^{\mu\nu}(p) = \mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{\text{Tr}[\gamma^\mu (\not{k} - \not{p} + M_b) \gamma^\nu (\not{k} + M_a)]}{[(k-p)^2 - M_b^2](k^2 - M_a^2)}. \tag{4.2}$$

The coefficients in the second line of eq. (4.1) are

$$\begin{aligned}
a_1 &= \frac{3}{2} g^2 (M_a - M_b)^2 - \frac{N_c}{2\pi^2} \int_0^1 dx \ln \frac{M_a^2(1-x) + M_b^2 x}{m^2} [M_a^2(1-x) + M_b^2 x - M_a M_b], \\
a_2 &= g^2 - \frac{N_c}{2\pi^2} \int_0^1 dx x(1-x) \left[1 - \frac{M_a M_b}{M_a^2(1-x) + M_b^2 x} + 2 \ln \frac{M_a^2(1-x) + M_b^2 x}{m^2} \right], \\
b_2 &= g^2 - \frac{N_c}{\pi^2} \int_0^1 dx x(1-x) \ln \frac{M_a^2(1-x) + M_b^2 x}{m^2}, \\
a_3(p^2) &= \frac{N_c}{\pi^2} \int_0^1 dx \left[x(1-x) \ln \left(1 - \frac{p^2 x(1-x)}{M_a^2(1-x) + M_b^2 x} \right) - \frac{R(p^2)}{2p^2} [M_a^2(1-x) + M_b^2 x - M_a M_b] \right], \\
b_3(p^2) &= \frac{N_c}{\pi^2} \int_0^1 dx x(1-x) \ln \left(1 - \frac{p^2 x(1-x)}{M_a^2(1-x) + M_b^2 x} \right), \\
R(p^2) &= \ln \left(1 - \frac{p^2 x(1-x)}{M_a^2(1-x) + M_b^2 x} \right) + \frac{p^2 x(1-x)}{M_a^2(1-x) + M_b^2 x}.
\end{aligned} \tag{4.3}$$

Because $R(p^2)$ is of $O(p^4)$, we find that $a_3(p^2) \sim O(p^2)$, $b_3(p^2) \sim O(p^2)$. The constant g in eq. (4.3) is a universal coupling constant, which absorbs the logarithmic divergence originating from quark loop integral,

$$\frac{3}{8} g^2 = \frac{N_c}{(4\pi)^{d/2}} \left(\frac{\mu^2}{m^2} \right)^{\epsilon/2} \Gamma \left(2 - \frac{d}{2} \right). \tag{4.4}$$

At chiral limit ($m_u = m_d = m_s = 0$), we can find that

$$\begin{aligned}
a_1 &= 0, \\
a_2 &= b_2 = g^2, \\
a_3(p^2) &= b_3(p^2) = \frac{N_c}{\pi^2} \int_0^1 dx x(1-x) \ln \left(1 - \frac{p^2 x(1-x)}{m^2} \right).
\end{aligned} \tag{4.5}$$

Therefore, S_{kin} is close to the standard form

$$S_{\text{kin}}^{\text{stand}} = \frac{1}{4} \sum_{ab} \int \frac{d^4 p}{(2\pi)^4} V_\mu^{ab}(p) V_\nu^{ba}(-p) [-g^{\mu\nu} p^2 + p^\mu p^\nu + m_{\nu ab}^2 g^{\mu\nu}], \tag{4.6}$$

provided we rescale $V_\mu \rightarrow V_\mu / \sqrt{a_2} = V_\mu / g$. While beyond the chiral limit, $a_2 \neq b_2$, so the $-g^{\mu\nu} p^2$ term and the $p^\mu p^\nu$ term have no common coefficients. Gauge symmetry is thus broken. But we don't need to bother about this, because throughout this paper, the condition $\partial_\mu V^\mu = 0$ is used. The $p^\mu p^\nu$ term can thus be discarded.

Another problem is the higher derivatives in eq. (4.1), which makes the vector mesons ill-defined. Fortunately, because the vector meson field is external, we can make a momentum-dependent transformation $V_\mu(p) \rightarrow (1 + f(p^2)p^2)V_\mu(p)$ such that, in the final form, terms with derivatives higher than 2 vanish, i.e.

$$a_3(p^2) - 2f_1a_1 + (a_1 - a_2p^2 + a_3(p^2)p^2)(2f(p^2) + f^2(p^2)p^2) = 0, \quad (4.7)$$

where $f_1 = f(p^2)|_{p^2=0}$. If $a_1 \neq 0$ (i.e. $m_a \neq m_b$), eq. (4.7) can't determine the value of f_1 . However, when $m_a = m_b$, we have $a_1 = 0$, and f_1 can be determined as $f_1 = a_3^1/(2a_2)|_{m_a=m_b}$, where $a_3^1 = a_3(p^2)|_{p^2=0}$. In the case of $m_a \neq m_b$, because a_1 is very small, f_1 changes slightly. $a_3^1/(2a_2)|_{m_a=m_b}$ is thus a good approximation for f_1 at this case. Substituting $(m_a + m_b)/2$ for the argument m_a in this expression, we obtain

$$f_1 = -N_c \left\{ 15\pi^2(M_a + M_b)^2 \left[g^2 - \frac{N_c}{6\pi^2} \ln \frac{(M_a + M_b)^2}{4m^2} \right] \right\}^{-1}. \quad (4.8)$$

Then we get the transformation factor

$$\alpha(p^2) \equiv 1 + f(p^2)p^2 = \sqrt{1 + \frac{(2f_1a_1 - a_3(p^2))p^2}{a_1 - a_2p^2 + a_3(p^2)p^2}}. \quad (4.9)$$

After this transformation, we find that

$$S_{\text{kin}} = \frac{1}{4} \sum_{ab} \int \frac{d^4p}{(2\pi)^4} V_\mu^{ab}(p) V^{\mu ba}(-p) (a_1 - a_2p^2 + 2f_1a_1p^2). \quad (4.10)$$

Comparing it with the standard form, we find the proper rescaling

$$V_\mu \rightarrow \frac{V_\mu}{\tilde{g}} \equiv \frac{V_\mu}{\sqrt{a_2 - 2f_1a_1}}. \quad (4.11)$$

The physical pseudoscalar meson fields can be obtained via field rescaling $\Phi \rightarrow 2F_\Phi^{-1}\Phi$ ($\Phi = \pi, K, \eta_8$).

What should be noted is that, the rescaling factor $1/\tilde{g}$ and the transformation factor $\alpha(p^2)$ are both flavor-related. They are different for different vector mesons (for ρ , the rescaling factor is $1/\tilde{g} = 1/g$). In the following expressions, We shall omit factors as $1/\tilde{g}$, $\alpha(p^2)$ and $2/F_\Phi$ for external lines, and include them only in final results.

V. QUARK LOOP CONTRIBUTIONS TO VECTOR MESON DECAYS

A. Effective actions

For vector mesons decays, we should include the two-point and three-point diagram for tree level of effective actions. The space-like condition of vector meson $\partial^\mu V_\mu = 0$ is used to simplify the calculations.

For two-point diagram, we should calculate fig.2(a). The corresponding effective action reads

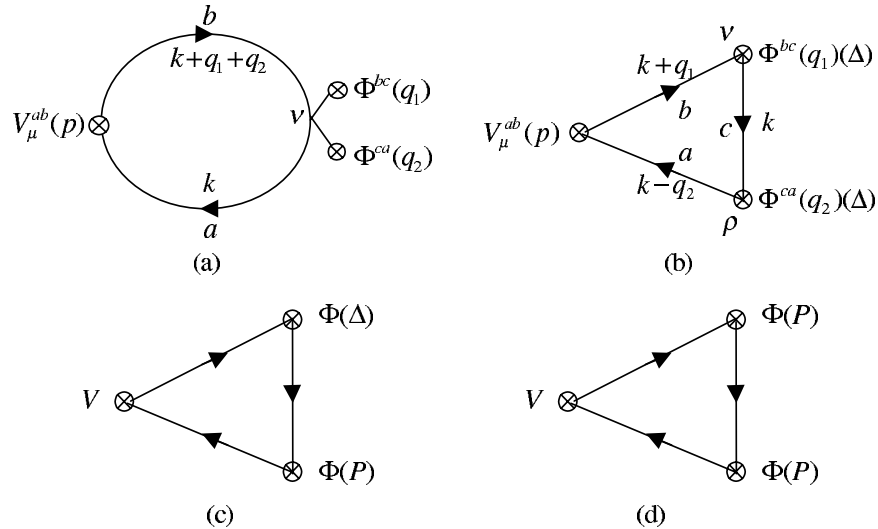


FIG. 2. Two-point and three-point diagrams for effective actions.

$$\begin{aligned}
S_2 &= \frac{iN_c}{8} \sum_{abc} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) (q_2 - q_1)_\nu T_{ab}^{\mu\nu}(q_1 + q_2) \\
&= \sum_{abc} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) q_2^\mu A_{ab}(p^2),
\end{aligned} \tag{5.1}$$

where

$$\begin{aligned}
A_{ab}(p^2) &= \frac{3}{16} g^2 (M_a - M_b)^2 - \frac{1}{8} g^2 p^2 - \frac{N_c}{16\pi^2} \int_0^1 dx \ln \frac{D_1^{ab}}{m^2} [M_a^2(1-x) + M_b^2 x - M_a M_b - 2x(1-x)p^2], \\
D_1^{ab}(p^2) &= M_a^2(1-x) + M_b^2 x - p^2 x(1-x).
\end{aligned} \tag{5.2}$$

Contributions from S' vanishes here.

For three-point diagram, we should include three kinds of vertices: $V - \Delta\Delta, V - \Delta P, V - PP$. The contribution from the first vertex (fig.2(b)) is

$$\begin{aligned}
S_3^\Delta &= \frac{i}{4} N_c g_A^2 \sum_{abc} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) q_{1\nu} q_{2\rho} T_{ab-c}^{\mu\nu\rho}(q_1, q_2) \\
&= \sum_{abc} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) q_2^\mu g_A^2 B_{ab-c}(p^2, q_1^2, q_2^2),
\end{aligned} \tag{5.3}$$

Here, we have used the definitions that

$$\begin{aligned}
T_{abc}^{\mu\nu\rho}(q_1, q_2) &= \mu^\epsilon \int \frac{d^d k}{(2\pi)^d} \frac{Tr[\gamma^\mu(\not{k} + \not{q}_1 + M_b) \gamma^\nu(\not{k} + M_c) \gamma^\rho(\not{k} - \not{q}_2 + M_a)]}{[(k + q_1)^2 - M_b^2](k^2 - M_c^2)[(k - q_2)^2 - M_a^2]}, \\
T_{ab-c}^{\mu\nu\rho} &= T_{abc}^{\mu\nu\rho}(M_c \rightarrow -M_c).
\end{aligned} \tag{5.4}$$

and

$$\begin{aligned}
B_{abc}(p^2, q_1^2, q_2^2) &= -\frac{1}{8} \left[\left(\frac{N_c}{6\pi^2} - g^2 \right) p^2 + \frac{N_c}{2\pi^2} \int_0^1 x dx \int_0^1 dy \left(\alpha_1 \ln \frac{D_2^{abc}}{m^2} + \frac{\alpha_2^{abc}}{D_2^{abc}} \right) \right], \\
\alpha_1 &= p^2(1-xy) - q_1^2(1-x-xy) - q_2^2 x(1-2y), \\
\alpha_2^{abc} &= -p^2 M_c [M_a x(1-y) + M_b(1-x)] + q_1^2 M_a [M_b(1-x) - M_c x y] + q_2^2 M_b x [M_a(1-y) - M_c y] \\
&\quad + p^2 q_1^2 x(1-x)^2 + p^2 q_2^2 x^2(1-x+xy)(1-y)^2 + q_1^2 q_2^2 x^2 y^2(1-xy), \\
D_2^{abc} &= M_a^2 x(1-y) + M_b^2(1-x) + M_c^2 x y - p^2 x(1-x)(1-y) - q_1^2 x y(1-x) - q_2^2 x^2 y(1-y), \\
B_{ab-c} &= B_{abc}(M_c \rightarrow -M_c).
\end{aligned} \tag{5.5}$$

The contributions from the latter two vertices (fig.2(c) and (d)) are

$$S_3^P = \sum_{abc} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) q_2^\mu B'_{abc}(p^2, q_1^2, q_2^2), \tag{5.6}$$

where B' is defined as

$$\begin{aligned}
V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) B'_{abc}(p^2, q_1^2, q_2^2) &= \frac{\kappa m_s N_c}{(4\pi)^2} \int_0^1 x dx \int_0^1 dy \\
&\quad \left\{ V_\mu^{ab}(p) \Phi^{bc}(q_1) \tilde{\Phi}^{ca}(q_2) g_A \left[M_c - M_a + M_b + \left(\frac{6\pi^2 g^2}{N_c} - \ln \frac{D_2^{abc}}{m^2} \right) (M_a - 2M_b - M_c) - \frac{\beta_1^{abc}}{D_2^{abc}} \right] \right. \\
&\quad + V_\mu^{ab}(p) \tilde{\Phi}^{bc}(q_1) \Phi^{ca}(q_2) g_A \left[M_a - M_b + M_c + \left(\frac{6\pi^2 g^2}{N_c} - \ln \frac{D_2^{abc}}{m^2} \right) (M_b - 2M_a - M_c) - \frac{\beta_2^{abc}}{D_2^{abc}} \right] \\
&\quad \left. + V_\mu^{ab}(p) \tilde{\Phi}^{bc}(q_1) \tilde{\Phi}^{ca}(q_2) \kappa m_s \left[1 + xy - (1 + 3xy) \left(\frac{6\pi^2 g^2}{N_c} - \ln \frac{D_2^{abc}}{m^2} \right) - \frac{\beta_3^{abc}}{D_2^{abc}} \right] \right\},
\end{aligned} \tag{5.7}$$

with

$$\begin{aligned}
\beta_1^{abc} &= M_a M_b M_c + p^2 x(1-y)[M_a x(1-y) + M_b(1-x) + M_c(1-x+xy)] \\
&\quad + q_1^2 xy[M_a(1-xy) + M_b(1-x) + M_c xy] + q_2^2 M_b x^2 y(1-y) \\
\beta_2^{abc} &= M_a M_b M_c + p^2(1-x)[M_a x(1-y) + M_b(1-x) + M_c x] \\
&\quad + q_1^2 M_a xy(1-x) + q_2^2 xy[M_a x(1-y) + M_b(1-xy) + M_c xy] \\
\beta_3^{abc} &= M_a M_b(1-xy) + (M_a + M_b)M_c xy + p^2 x(1-x)(1-y)(1+xy) + q_1^2 x^2 y^2(1-x) + q_2^2 x^3 y^2(1-y).
\end{aligned} \tag{5.8}$$

Here,

$$\tilde{\Phi} = \begin{pmatrix} 0 & 0 & \Phi^{13} \\ 0 & 0 & \Phi^{23} \\ \Phi^{31} & \Phi^{32} & 2\Phi^{33} \end{pmatrix} = \sqrt{2} \begin{pmatrix} 0 & 0 & K^+ \\ 0 & 0 & K^0 \\ K^- & \bar{K}^0 & -\frac{4}{\sqrt{6}}\eta_8 \end{pmatrix}, \tag{5.9}$$

which results from the simplified $\mathcal{M} = \text{diag}\{0, 0, m_s\}$. Note that the form of B' is not given directly, which depends on the flavor index. For ρ and ω decay, $B' = 0$.

Thus the total action from quark loop is

$$S_{\text{quark}} = \sum_{abc} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{ca}(q_2) q_2^\mu f_{abc}^{(0)}(p^2, q_1^2, q_2^2), \tag{5.10}$$

where

$$f_{abc}^{(0)}(p^2, q_1^2, q_2^2) = A_{ab}(p^2) + g_A^2 B_{ab-c}(p^2, q_1^2, q_2^2) + B'_{abc}(p^2, q_1^2, q_2^2). \tag{5.11}$$

Now we have resummed all orders of momentum expansion, which is embodied by D_1^{ab} and D_2^{abc} . To obtain the leading order of momentum expansion, we should first take chiral limit $m_u = m_d = m_s = 0$. But we should remember that, as indicated at the end of Sec.III, it is inconsistent to study ϕ physics at chiral limit.

B. Vector meson dominant and KSFR sum rules

We also care about decay $\rho \rightarrow e^+ e^-$. The direct coupling between photon and vector meson resonances is also yielded by the effects of quark loops. In chiral limit, the VMD vertex at the leading order of large N_c expansion, after the condition $\partial^\mu V_\mu = 0$ is applied, reads

$$S_{\text{VMD}} = -\frac{e}{2} \int \frac{d^4 p}{(2\pi)^4} A_\mu(-p) < Q V^\mu(p) > f_{\rho\gamma}^{(0)}(p^2) p^2, \tag{5.12}$$

where A^μ is photon field, $Q = \text{diag}\{2/3, -1/3, -1/3\}$ is charge operator of quark fields, and

$$f_{\rho\gamma}^{(0)}(p^2) = -\frac{8A_{11}(p^2)}{gp^2} = g - \frac{N_c}{g\pi^2} \int_0^1 dx \cdot x(1-x) \ln\left(1 - \frac{x(1-x)p^2}{m^2}\right). \tag{5.13}$$

In addition, at the leading order of large N_c expansion, the $V - \Phi\Phi$ vertex (where V stands for vector mesons and Φ for pseudoscalar mesons) in chiral limit reads from eq. (5.10)

$$S_{V\Phi\Phi} = -\frac{1}{2} \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) < V_\mu(p) \Phi(q_1) \Phi(q_2) > q_2^\mu f_{\rho\pi\pi}^{(0)}(p^2) p^2, \tag{5.14}$$

where

$$f_{\rho\pi\pi}^{(0)}(p^2) = -\frac{8f_{111}(p^2, 0, 0)}{gF_\pi^2 p^2}. \tag{5.15}$$

The rescaling factors $1/g$ for vector mesons and $2/F_\pi$ for pseudoscalar mesons is included both in eq. (5.13) and eq. (5.15).

It is well known that the KSFR(I) sum rule [22]

$$f_{\rho\gamma}(m_\rho^2) = f_{\rho\pi\pi}(m_\rho^2) F_\pi^2 \tag{5.16}$$

is the result of current algebra and PCAC. We expect it to be valid at the leading order of large N_c expansion. Therefore, the KSRF(I) sum rule is satisfied when $B_{11-1}(p^2, 0, 0)|_{p^2=m_\rho^2} = 0$, or, using definition (5.5),

$$g^2 = \frac{N_c}{2\pi^2} \int_0^1 dx \int_0^1 dy \cdot x(1-xy) \left[1 + \frac{m^2}{m^2 - x(1-x)(1-y)p^2} + \ln \frac{m^2 - x(1-x)(1-y)p^2}{m^2} \right] \Big|_{p^2=m_\rho^2}. \quad (5.17)$$

Setting $m \simeq 460\text{MeV}$ (see Appendix A), we find that $g = 0.329$.

VI. GOLDSTONE BOSON LOOP CONTRIBUTIONS TO VECTOR MESON DECAYS

In this section, we shall calculate one-loop correction of pseudoscalar meson to vector mesons decays and provide a complete prediction on these reactions. Because the contribution of loop effects is suppressed by N_c^{-1} expansion, we shall use some approximations for simplicity.

Firstly, because the quark masses in meson loop are doubly suppressed, we shall discard S' and κP parts in \mathcal{L}_χ^I , because they are of $O(m_s)$. This means $B' = 0$ when tree-level action is used in meson-loop calculations. Secondly, in propagators of pseudoscalar mesons, since $m_\pi^2 \ll m_K^2$, we assume that pion is a massless particle.

There is another approximation we have used. In dimensional regularization, $\int d^d k k^{2n} \equiv 0$ ($n \geq -1$). Thus, we have

$$\int d^d k \frac{k^{2s}}{k^2 - m_\Phi^2 + i\epsilon} \propto m_\Phi^{2s} \times \text{quadratically divergent term}, \quad (s \geq 1).$$

Therefore, in this paper we shall ignore all contributions from quartic divergences or higher order ones. As we shall see, in the following calculation the lowest order divergence is quadratic. It means that we can take the approximation $q^2 = 0$ (q^μ is the four-momentum of pseudoscalar mesons) in calculation on pseudoscalar meson loops.

At the order of N_c^{-1} expansion next to the leading one, there are three kinds of loop diagrams of pseudoscalar mesons need to be calculated (fig.3). As to the tadpole diagram in fig.3(a) and (d), we only include contributions from K and η_8 mesons, since the massless π meson does not contribute due to the relation $\int d^d k k^{2n} \equiv 0$ ($n \geq -1$). In addition, when we calculate two-point diagram (fig.3(b)(e)), we must include all chain-like diagrams of pseudoscalar meson loops which have imaginary part (fig.3(c)(f)), e.g., $\pi\pi$ loop for ρ decay. Because it will generate a large imaginary part of S -matrix for vector mesons physics.

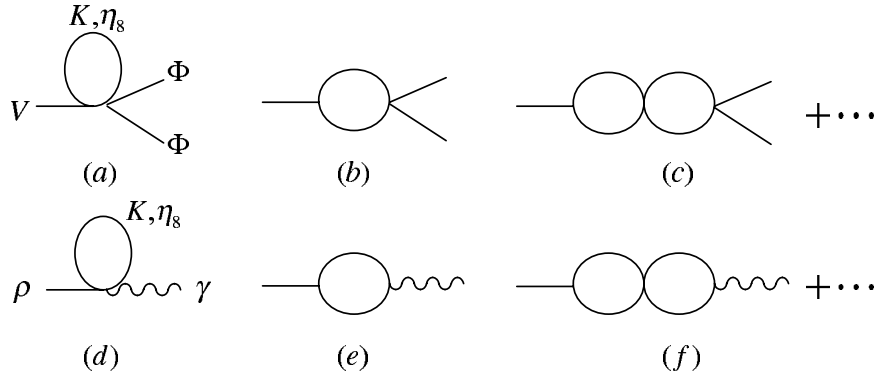


FIG. 3. One loop diagrams of Goldstone bosons for vertices $V - \Phi\Phi$ and $\rho - \gamma$. (a)(d)Tadpole diagram of K and η_8 . (b)(e)Two-point diagram of pseudoscalar mesons. (c)(f)Chain-like diagrams of pseudoscalar meson loops.

A. Tadpole Diagram

When calculating tadpole, we need various quark one-loop diagrams with various external vertices but with one vector meson external source and four pseudoscalar meson external sources. They are shown in fig.4.

As we can see later, the contributions from fig.(e) and (f) can be omitted, so we have

$$S_{V-4\Phi} = \sum_{abcde} \int \frac{d^4 p d^4 q_1 \cdots d^4 q_4}{(2\pi)^{4 \times 4}} \delta(p + q_1 + \cdots + q_4) V_\mu^{ab}(p) \Phi^{bc}(q_1) \Phi^{cd}(q_2) \Phi^{de}(q_3) \Phi^{ea}(q_4) f_{abcde}^\mu(q_1, \cdots, q_4), \quad (6.1)$$

where f^μ includes contributions from fig. (a)~(d):

$$\begin{aligned}
f_{abcde}^\mu(q_1, \dots, q_4) = & \frac{i}{2^7 \times 3} N_c (q_1 - 3q_2 + 3q_3 - q_4)_\nu T_{ab}^{\mu\nu} (q_1 + \dots + q_4) \\
& - \frac{i}{64} N_c (q_2 - q_1)_\nu (q_4 - q_3)_\rho T_{abd}^{\mu\nu\rho} (q_1 + q_2, q_3 + q_4) \\
& - \frac{i}{96} N_c g_A^2 [q_{1\nu} (q_2 - 2q_3 + q_4)_\rho T_{ab-c}^{\mu\nu\rho} (q_1, q_2 + q_3 + q_4) \\
& + (q_1 - 2q_2 + q_3)_\nu q_{4\rho} T_{ab-e}^{\mu\nu\rho} (q_1 + q_2 + q_3, q_4)].
\end{aligned} \tag{6.2}$$

Here, we have omitted S' and κP in \mathcal{L}_χ^I .

Now, arbitrary two of the four Φ should be contracted as long as they are η_8 or K . Using the propagators for them in momentum space such as

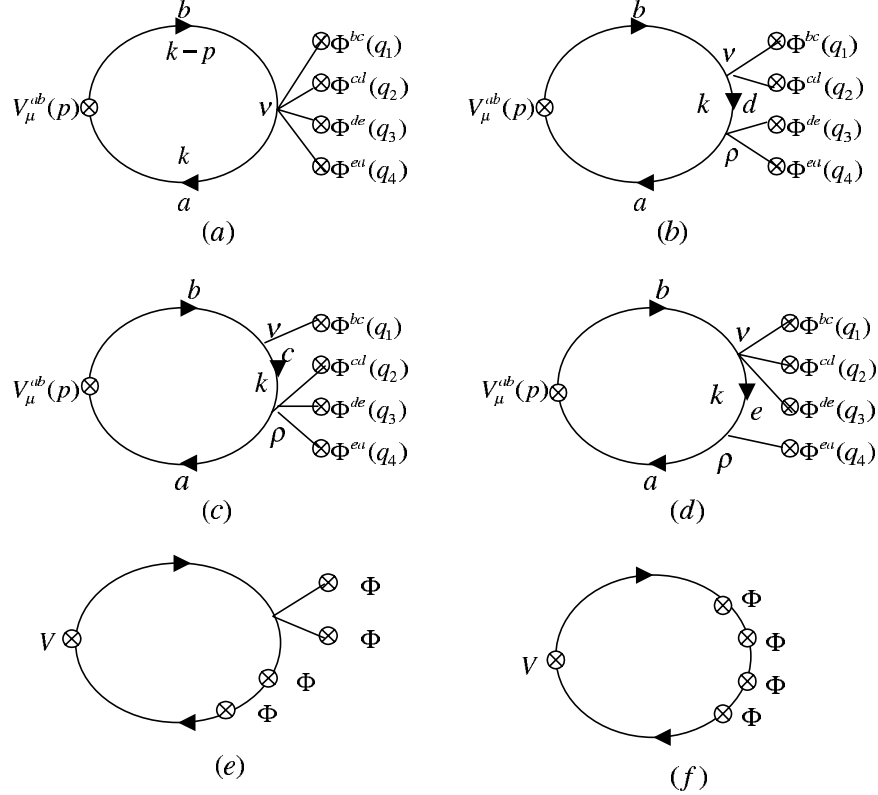


FIG. 4. Diagrams of quark loop for vertex $V = 4\Phi$.

$$\overline{K^+(q_1)K^-(q_2)} = (2\pi)^4 \delta(q_1 + q_2) \frac{i}{q_1^2 - m_K^2 + i\epsilon}, \tag{6.3}$$

(remembering $m_K = m_{\eta_8}$) and considering all possible contractions, we have

$$\begin{aligned}
S_{\text{tad}} = & 2 \int \frac{d^4 p d^4 q_1 d^4 q_2}{(2\pi 2\pi)^4} \delta(p + q_1 + q_2) V_\mu^{13}(p) \Phi^{32}(q_1) \Phi^{21}(q_2) \int \frac{d^4 q}{(2\pi)^4} \frac{i}{q^2 - m_K^2} \\
& \left[\frac{1}{6} f_{13211}^\mu(q_1, q_2, q, -q) + f_{13213}^\mu(q_1, q_2, q, -q) + \frac{1}{6} f_{13221}^\mu(q_1, q, q_2, -q) + \frac{1}{6} f_{13222}^\mu(q_1, q, q_2, -q) \right. \\
& + f_{13232}^\mu(q_1, q, -q, q_2) - \frac{2}{6} f_{13321}^\mu(q, q_1, q_2, -q) - \frac{2}{6} f_{13322}^\mu(q, q_1, -q, q_2) + 2 f_{13132}^\mu(q, -q, q_1, q_2) \\
& \left. + \left(\frac{2}{\sqrt{6}} \right)^2 f_{13332}^\mu(q, -q, q_1, q_2) \right]
\end{aligned} \tag{6.4}$$

for the vertex $K^{*+}(p)\bar{K}^0(q_1)\pi^-(q_2)$ and similar expressions for the vertices $\rho(p)\pi(q_1)\pi(q_2)$ and $\phi(p)K(q_1)K(q_2)$. When expanding the expression in the brackets in powers of q^μ , and discarding the odd-order terms, we obtain a polynomial

in powers of q^2 . Because the two contracted Φ s are external fields as far as the quark loop concerned, due to the approximation $q^2 = 0$, what we do amounts to setting $q_\mu = 0$ in the bracket. (Now let us turn to the contribution of fig.(e) to f^μ . It must be of this form: $(q_2 - q_1)_\nu q_{3\rho} q_{4\sigma} T_{abcd}^{\mu\nu\rho\sigma}(q_1 + q_2, q_3, q_4)$. After contraction, two of the four q s should be set to be 0. This term is thus vanished. Similar argument is valid for fig.(f).) The calculations are thus simplified. Eventually, we get

$$S_{\text{tad}} = \int \frac{d^4 p d^4 q}{(2\pi 2\pi)^4} V_\mu^{ab}(p) \Phi^{bc}(-p - q) \Phi^{ca}(q) q^\mu C_{abc}(p^2) \quad (6.5)$$

with

$$\begin{aligned} C_{122}(p^2) &= \frac{\lambda m_K^2}{6(4\pi)^2 F_K^2} [3B_{113}(p^2) - 3g_A^2 B_{11-1}(p^2) - 5f_{111}^{(0)}(p^2)] \\ C_{132}(p^2) &= \frac{\lambda m_K^2}{6(4\pi)^2 F_K^2} [3B_{133}(p^2) - 3g_A^2 B_{13-1}(p^2) - 10f_{131}^{(0)}(p^2)] \\ C_{33c}(p^2) &= \frac{3\lambda m_K^2}{4(4\pi)^2 F_K^2} [B_{331}(p^2) - g_A^2 B_{33-1}(p^2) - 3f_{331}^{(0)}(p^2)] \quad (c = 1, 2) \end{aligned} \quad (6.6)$$

for vertex $\rho^+ \pi^0 \pi^-$, $K^{*+} \bar{K}^0 \pi^-$ and $\phi K K$ respectively, where $B(p^2) = B(p^2, 0, 0)$, and

$$f_{abc}^{(0)}(p^2) = A_{ab}(p^2) + g_A^2 B_{ab-c}(p^2) \quad (6.7)$$

is the simplified form of $f_{abc}^{(0)}(p^2, q_1^2, q_2^2)$ according to $q^2 = 0$. Here, we have neglected the differences between m_K and m_η , and between F_K and F_η , since they are doubly suppressed by light quark mass expansion and N_c^{-1} expansion. The constant λ absorbs the divergence:

$$\lambda = \left(\frac{4\pi\mu^2}{m_K^2} \right)^{\epsilon/2} \Gamma \left(1 - \frac{d}{2} \right). \quad (6.8)$$

After similar consideration, we can find the tadpole loop corrections of K or η_8 mesons to VMD vertex (see fig.3(d)) is

$$S_{\text{VMD}}^{(t)} = \frac{e\lambda m_K^2}{16\pi^2 F_K^2} \int \frac{d^4 p}{(2\pi)^4} A_\mu(-p) \rho^{0\mu}(p) f_{\rho\gamma}^{(0)}(p^2) p^2. \quad (6.9)$$

In Appendix B, $\lambda = 0.54$ is determined.

B. Two-Point Diagram and Chain-Like Approximation

For two-point diagram, we need the vertex of 4Φ , which in principle should be generated from quark loop. For simplicity, we can alternatively obtain it from the effective Lagrangian of ChPT at order p^2 and order p^4 ,

$$\begin{aligned} \mathcal{L}_{4\Phi} &= \frac{F_0^2}{96} (< \partial_\mu \Phi \Phi \partial^\mu \Phi \Phi > - < \partial_\mu \Phi \partial^\mu \Phi \Phi^2 >) \\ &+ L_1 < \partial_\mu \Phi \partial^\mu \Phi > < \partial_\nu \Phi \partial^\nu \Phi > + L_2 < \partial_\mu \Phi \partial_\nu \Phi > < \partial^\mu \Phi \partial^\nu \Phi > + L_3 < \partial_\mu \Phi \partial^\mu \Phi \partial_\nu \Phi \partial^\nu \Phi >, \end{aligned} \quad (6.10)$$

with the coefficients determined by ChQM (see Appendix A)

$$L_1 = \frac{\gamma}{24}, \quad L_2 = \frac{\gamma}{12}, \quad L_3 = \frac{\gamma}{12}(g_A^4 - 3), \quad \gamma = \frac{N_c}{(4\pi)^2}. \quad (6.11)$$

At chiral limit, it is known that the renormalized F_0 is just the decay constant of π mesons: $F_0 = F_\pi$.

As to the vertex $V - \Phi\Phi$, it is the sum of two- and three-point diagrams of quark loops:

$$\mathcal{L}_{V-2\Phi} = i \sum_{abc} f_{abc}^{(0)} (-\nabla^2) V_\mu^{ab}(x) \Phi^{bc}(x) \partial^\mu \Phi^{ca}(x), \quad (6.12)$$

where ∇^2 acts only on the coordinates of V_μ . Thus, the effective Lagrangian is $\mathcal{L}_{V-2\Phi} + \mathcal{L}_{4\Phi}$. Contracting two Φ s in $\mathcal{L}_{V-2\Phi}$ with any two Φ s in $\mathcal{L}_{4\Phi}$, and summing over all possible contractions of K , η_8 and π (see fig.3(b)), we have

$$S_{\text{TPD}} = \sum_{abc} \int \frac{d^4 p d^4 q}{(2\pi^2 \pi)^4} V_\mu^{ab}(p) \Phi^{bc}(-p-q) \Phi^{ca}(q) q^\mu E_{abc}(p^2), \quad (6.13)$$

with

$$\begin{aligned} E_{122}(p^2) &= f_{111}^{(0)}(p^2) \Sigma_\pi(p^2) + f_{113}^{(0)}(p^2) \Sigma_K(p^2) \\ E_{132}(p^2) &= f_{131}^{(0)}(p^2) \Sigma_{K\pi}(p^2) + \frac{1}{2} \left(f_{131}^{(0)}(p^2) + 2f_{133}^{(0)}(p^2) \right) \Sigma_K(p^2) \\ E_{33c}(p^2) &= 3f_{331}^{(0)}(p^2) \Sigma_K(p^2) \end{aligned} \quad (6.14)$$

for vertex $\rho^+ \pi^0 \pi^-$, $K^{*+} \bar{K}^0 \pi^-$ and $\phi K K$ respectively, where

$$\begin{aligned} \Sigma_\pi(p^2) &= -\frac{F_0^2 p^2}{4\pi^2 F_\pi^4} \left(1 + \frac{3p^2}{4F_0^2} (g^2 - \frac{g_A^4}{3\pi^2}) \right) \left[\frac{\lambda}{6} + \int_0^1 dx x(1-x) \ln \frac{x(1-x)p^2}{m_K^2} \right. \\ &\quad \left. + \frac{i}{6} \text{Arg}(-1) \theta(p^2 - 4m_\pi^2) \left(1 - \frac{4m_\pi^2}{p^2} \right)^{3/2} \right] \\ \Sigma_K(p^2) &= \frac{F_0^2 m_K^2}{8\pi^2 F_K^4} \left(1 + \frac{3p^2}{4F_0^2} (g^2 - \frac{g_A^4}{3\pi^2}) \right) \left[\lambda \left(1 - \frac{p^2}{6m_K^2} \right) + \int_0^1 dx \left(1 - x(1-x) \frac{p^2}{m_K^2} \right) \ln \left(1 - x(1-x) \frac{p^2}{m_K^2} \right) \right] \\ &= \frac{F_0^2 m_K^2}{8\pi^2 F_K^4} \left(1 + \frac{3p^2}{4F_0^2} (g^2 - \frac{g_A^4}{3\pi^2}) \right) \left[\lambda \left(1 - \frac{p^2}{6m_K^2} \right) + \int_0^1 dx \left(1 - x(1-x) \frac{p^2}{m_K^2} \right) \ln \left| 1 - x(1-x) \frac{p^2}{m_K^2} \right| \right. \\ &\quad \left. - \frac{i}{6} \text{Arg}(-1) \theta(p^2 - 4m_K^2) \frac{p^2}{m_K^2} \left(1 - \frac{4m_K^2}{p^2} \right)^{3/2} \right] \\ \Sigma_{K\pi}(p^2) &= \frac{3F_0^2 m_K^2}{16\pi^2 F_K^2 F_\pi^2} \left(1 + \frac{3p^2}{4F_0^2} (g^2 - \frac{g_A^4}{3\pi^2}) \right) \left[\frac{\lambda}{2} \left(1 - \frac{p^2}{3m_K^2} \right) + \int_0^1 dx \left(x - x(1-x) \frac{p^2}{m_K^2} \right) \ln \left| x - x(1-x) \frac{p^2}{m_K^2} \right| \right. \\ &\quad \left. - \frac{i}{6} \text{Arg}(-1) \theta(p^2 - (m_\pi + m_K)^2) \frac{p^2}{m_K^2} \left(1 - \frac{2(m_K^2 + m_\pi^2)}{p^2} + \frac{(m_K - m_\pi)^4}{p^4} \right)^{3/2} \right] \end{aligned} \quad (6.15)$$

and

$$\begin{aligned} \text{Arg}(-1) &= (1 + 2k)\pi, & k &= 0, \pm 1, \pm 2, \dots, \\ \theta(x - y) &= \begin{cases} 1, & x > y; \\ 0, & x \leq y. \end{cases} \end{aligned} \quad (6.16)$$

Note that $\Sigma_K(p^2)$ has no imaginary part for $E_{122}(p^2)$ and $E_{132}(p^2)$, while has imaginary part for $E_{33c}(p^2)$, because $m_\rho, m_{K^*} < 2m_K$ while $m_\phi > 2m_K$. As we can see, two-point diagrams give an imaginary contribution, which results from propagator of lighter pseudoscalar mesons. In $\Sigma_\pi(p^2)$ and $\Sigma_{K\pi}(p^2)$, we have discarded m_π in the real part, leaving it in the imaginary part, because it becomes important there. We also have neglected the unimportant difference between K and η_8 in $E_{132}(p^2)$. There is no $\eta_8 - \eta_8$ loop contribution in E_{33c} , because it is constrained by space-like condition of external vector mesons.

Considering all chain-like loop diagrams of complex pseudoscalar loops, i.e., Σ_π , $\Sigma_{K\pi}$ and Σ_K for ρ , K^* and ϕ decay respectively, we have

$$S_{\text{chain}} = \sum_{abc} \int \frac{d^4 p d^4 q}{(2\pi^2 \pi)^4} V_\mu^{ab}(p) \Phi^{bc}(-p-q) \Phi^{ca}(q) q^\mu E'_{abc}(p^2), \quad (6.17)$$

with

$$\begin{aligned} E'_{122}(p^2) &= f_{111}^{(0)}(p^2) \frac{\Sigma_\pi(p^2)}{1 - \Sigma_\pi(p^2)} + f_{113}^{(0)}(p^2) \Sigma_K(p^2) \\ E'_{132}(p^2) &= f_{131}^{(0)}(p^2) \frac{\Sigma_{K\pi}(p^2)}{1 - \Sigma_{K\pi}(p^2)} + \frac{1}{2} \left(f_{131}^{(0)}(p^2) + 2f_{133}^{(0)}(p^2) \right) \Sigma_K(p^2) \\ E'_{33c}(p^2) &= 3f_{331}^{(0)}(p^2) \frac{\Sigma_K(p^2)}{1 - \Sigma_K(p^2)} \end{aligned} \quad (6.18)$$

Similar consideration can also be applied to two-point diagram correction to VMD vertex $\rho - \gamma$. The $\gamma - \Phi\Phi$ vertex (for quark loops see fig.2(a) and (b), with V_μ replaced by A_μ) is

$$S_{\gamma-\Phi\Phi} = -e \sum_{abc} \int \frac{d^4 p d^4 q}{(2\pi 2\pi)^4} A_\mu(p) Q^{ab} \Phi^{bc}(-p-q) \Phi^{ca}(q) q^\mu \left(\frac{F_0^2}{4} - f_{abc}^{(0)} \right). \quad (6.19)$$

Combining it with $S_{\rho-\Phi\Phi}$ (see eq. (5.14)), we obtain, after contractions and chain-like approximation, pseudoscalar meson loop diagram correction (see fig.3(e)(f)) to VMD vertex $\rho - \gamma$

$$S_{\rho-\gamma}^{\text{chain}} = -\frac{e}{2} \int \frac{d^4 p}{(2\pi)^4} \rho_\mu^0(p) A^\mu(-p) E'_\gamma(p^2), \quad (6.20)$$

with

$$E'_\gamma(p^2) = -8 \left(\frac{f_{111}^{(0)}(p^2)}{F_\pi^2} \frac{\Sigma_\pi^\gamma(p^2)}{1 - \Sigma_\pi(p^2)} + \frac{f_{113}^{(0)}(p^2)}{F_K^2} \Sigma_K^\gamma(p^2) \right) \quad (6.21)$$

where

$$\begin{aligned} \Sigma_\pi^\gamma(p^2) &= -\frac{p^2(F_0^2 - 4f_{111}^{(0)}(p^2))}{4\pi^2 F_\pi^2} \left[\frac{\lambda}{6} + \int_0^1 dx x(1-x) \ln \frac{x(1-x)p^2}{m_K^2} + \frac{i}{6} \text{Arg}(-1) \theta(p^2 - 4m_\pi^2) \left(1 - \frac{4m_\pi^2}{p^2}\right)^{3/2} \right] \\ \Sigma_K^\gamma(p^2) &= \frac{m_K^2(F_0^2 - 4f_{113}^{(0)}(p^2))}{8\pi^2 F_K^2} \left[\lambda \left(1 - \frac{p^2}{6m_K^2}\right) + \int_0^1 dx \left(1 - x(1-x) \frac{p^2}{m_K^2}\right) \ln \left(1 - x(1-x) \frac{p^2}{m_K^2}\right) \right] \end{aligned} \quad (6.22)$$

VII. NUMERICAL RESULTS

The widths for on-shell decays of vector mesons are determined by

$$\begin{aligned} \Gamma_{\rho \rightarrow e^+ e^-} &= \frac{\pi \alpha_{e.m.}^2}{3} |\alpha(m_\rho^2) f_{\rho\gamma}^{(c)}(m_\rho^2)|^2 m_\rho, \\ \Gamma_{\rho \rightarrow \pi\pi} &= \frac{4}{3\pi} \left| \frac{\alpha(m_\rho^2) f_{111}^{(c)}(m_\rho^2, m_\pi^2, m_\pi^2)}{g F_\pi^2} \right|^2 m_\rho \left[1 - 4 \frac{m_\pi^2}{m_\rho^2} \right]^{3/2}, \\ \Gamma_{K^* \rightarrow K\pi} &= \frac{1}{\pi} \left| \frac{\alpha(m_{K^*}^2) f_{132}^{(c)}(m_{K^*}^2, m_K^2, m_\pi^2)}{\tilde{g} F_K F_\pi} \right|^2 m_{K^*} \left[1 - 2 \frac{m_K^2 + m_\pi^2}{m_{K^*}^2} + \frac{(m_K^2 - m_\pi^2)^2}{m_{K^*}^4} \right]^{3/2}, \\ \Gamma_{\phi \rightarrow KK} &= \frac{2}{3\pi} \left| \frac{\alpha(m_\phi^2) f_{331}^{(c)}(m_\phi^2, m_K^2, m_K^2)}{\tilde{g} F_K^2} \right|^2 m_\phi \left[1 - 4 \frac{m_K^2}{m_\phi^2} \right]^{3/2}. \end{aligned} \quad (7.1)$$

where $f_{\rho\gamma}^{(c)}$ gets contributions from eq. (5.12), (6.9), and (6.20), and $f_{abc}^{(c)}$ gets contributions from eq. (5.10), (6.5), and (6.13):

$$\begin{aligned} f_{\rho\gamma}^{(c)}(p^2) &= f_{\rho\gamma}^{(0)}(p^2) \left(1 - \frac{e\lambda m_K^2}{8\pi^2 F_K^2} \right) + E'_\gamma(p^2)/p^2, \\ f_{abc}^{(c)}(p^2, q_1^2, q_2^2) &= f_{abc}^{(0)}(p^2, q_1^2, q_2^2) + C_{abc}(p^2) + E'_{abc}(p^2). \end{aligned} \quad (7.2)$$

As indicated at the end of Sec.IV, we have included in eq. (7.1) the rescaling factor $1/\tilde{g} \equiv 1/\sqrt{a_2 - 2f_1 a_1}$ and the transformation factor $\alpha(p^2) \equiv 1 + f(p^2)p^2$ (which are both flavor-dependent) for vector meson, and rescaling factor $2/F_\pi, 2/F_K$ for external π, K mesons respectively. In last equation for ϕ decay, we should distinguish m_{K^\pm} from m_{K^0} when we consider $\phi \rightarrow K^+ K^-$ and $\phi \rightarrow K_L^0 K_S^0$ respectively, because this is important for the difference between the two decay widths.

Here are values of parameters: $m = 460\text{MeV}$ (chiral coupling constant at $O(p^4)$, see Appendix A), $m_s = 170\text{MeV}$ (input), $\kappa = 0.5$ (chiral coupling constant at $O(p^4)$), $\lambda = 0.54$ (Zweig rule), $g = 0.329$ (KSrf sum rule), $g_A = 0.75$ (β decay of neutron), $F_0 = F_\pi = 185\text{MeV}$, $F_K = 226\text{MeV}$, $m_\pi = 140\text{MeV}$, $m_{K^\pm} = 494\text{MeV}$, $m_{K^0} = 497.6\text{MeV}$, $m_\rho = 769\text{MeV}$, $m_{K^*} = 892\text{MeV}$, $m_\phi = 1019\text{MeV}$. The numerical results are listed in Table 1.

As we can see, the results after resummation are significantly different from the leading-order ones, which indicates that momentum expansion converges slowly. Therefore, the studies only based on the leading order or only up to the next order of momentum expansion are very incomplete. Resummation is definitely necessary here. Moreover, loop corrections of pseudoscalar mesons, which are next to the leading order of N_c expansion, play an important role, especially for SU(2) sector.

	Leading Order	Resummation	non-chiral limit $f^{(0)}(m_V^2, m_{\Phi_1}^2, m_{\Phi_2}^2)$	After Loop Correction $f^{(c)}(m_V^2, m_{\Phi_1}^2, m_{\Phi_2}^2)$	Experimental Value
$\rho \rightarrow e^+e^-$	0.00465	0.00654		0.00563	0.00677 ± 0.00032
$\rho \rightarrow \pi\pi$	123	187	194	175	150.8 ± 2.0
$K^* \rightarrow K\pi$	32.1	40.1	53.0	50.9	50.7 ± 0.9
$\phi \rightarrow K^+K^-$	$^\dagger 0.951$	$^\dagger 1.494$	2.410	2.147	2.193 ± 0.047
$\phi \rightarrow K_L^0 K_S^0$	$^\dagger 0.642$	$^\dagger 1.010$	1.643	1.465	1.507 ± 0.038

TABLE I. Numerical results for vector mesons decays. These value are in unit of MeV. The "leading order" and "resummation" columns show results of momentum expansion obtained at chiral limit. † These four values are listed just for uniformity and can not be treated seriously, because ϕ physics should be studied at non-chiral limit, which is required by unitarity. The two fs in second line are not for $\rho \rightarrow e^+e^-$ decay.

Unitarity condition of S -matrix, or optical theorem,

$$\text{Im}\mathcal{T}_{\beta,\alpha} = \frac{1}{2} \sum_{\text{all } \gamma} \mathcal{T}_{\gamma,\alpha} \mathcal{T}_{\gamma,\beta}^*, \quad (8.1)$$

has to be satisfied for any well-defined quantum field theory, where the $\mathcal{T}_{\beta,\alpha}$ is transition amplitude from state α to state β , and γ denotes all possible intermediate states on mass shells. It is well-known that a low energy effective meson theory should be a well-defined perturbative theory in N_c^{-1} expansion [25]. Therefore, we can expand \mathcal{T} -matrix in powers of N_c^{-1} ,

$$\mathcal{T} = \sum_{n=0}^{\infty} \mathcal{T}_n, \quad \mathcal{T}_n \sim O((N_c^{-\frac{1}{2}})^n). \quad (8.2)$$

Then the unitarity condition of S -matrix for a low energy effective meson theory has to be satisfied order by order in powers of N_c^{-1} ,

$$\text{Im}(\mathcal{T}_{\beta,\alpha})_n = \frac{1}{2} \sum_{\text{all } \gamma(m \leq n)} (\mathcal{T}_{\gamma,\alpha})_m (\mathcal{T}_{\gamma,\beta}^*)_{n-m}. \quad (8.3)$$

Now turn to the unitarity condition of S -matrix in the effective meson field theory deduced from ChQM. First, from effective action S_n we can see that every vertex is of $O(N_c)$. From eq.(4.4) it can be showed that $g \sim O(\sqrt{N_c})$. Moreover, eq.(3.5) shows that there is a term $(F_\pi^2/16) \int d^4x < \nabla_\mu U \nabla^\mu U^\dagger >$ (after renormalized) in S_2 , which means that $F_\pi \sim O(\sqrt{N_c})$. Because vector meson V and pseudoscalar meson Φ should be rescaled through $V \rightarrow V/g$ and $\Phi \rightarrow \Phi/F_\pi$ respectively, there is no difference between them as far as power counting about N_c is concerned. In what follows, the word "meson" means vector or pseudoscalar meson if not indicated. So in any Feynman diagram every external meson line is of $O(1/\sqrt{N_c})$ and every internal meson line is of $O(1/N_c)$. Therefore, any transition amplitudes with n_V vertex, n_e external meson lines, n_i internal meson lines and n_l loops of mesons are of order

$$N_c^{1-n_l-n_e/2} = (N_c^{-\frac{1}{2}})^{2n_l+n_e-2}, \quad (8.4)$$

where relation $n_l = n_i - n_V + 1$ has been used.

Now consider transition amplitude from n mesons state $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ to k mesons state $\{\beta_1, \beta_2, \dots, \beta_k\}$. Assuming γ is s mesons state $\{\gamma_1, \gamma_2, \dots, \gamma_s\}$, and using the power counting rule (8.4), eq.(8.3) can be written

$$\text{Im}(\mathcal{T}_{\beta,\alpha})_{(2n_l+k+n-2)} = \frac{1}{2} \sum_{\text{all } \gamma(s)} (\mathcal{T}_{\gamma,\alpha})_{(2n'_l+s+n-2)} (\mathcal{T}_{\gamma,\beta}^*)_{(2n''_l+s+k-2)}, \quad (8.5)$$

where n_l , n'_l and n''_l are meson loop numbers of transition amplitude $\mathcal{T}_{\beta,\alpha}$, $\mathcal{T}_{\gamma,\alpha}$ and $\mathcal{T}_{\gamma,\beta}$ respectively. Both side of eq. (8.5) should be of the same order, thus

$$n'_l + n''_l + s = 1 + n_l. \quad (8.6)$$

At the leading order of transition $\alpha \rightarrow \beta$, we have $n_l = 0$, so $n'_l = n''_l = 0$ and $s = 1$. It means that when summing over states γ in eq. (8.5), only one meson state should be included.

What interests us is meson decay, i.e., α is one meson state and $n = 1$. Then $s = 1$ indicates that only solution $\gamma = \alpha$ is allowed at the leading order. However, for $n'_l = 0$, $(\mathcal{T}_{\alpha,\alpha})_0 \equiv 0$, since meson fields are free point-particle at limit $N_c \rightarrow \infty$ [25]. Therefore, we have proved a theorem that on-shell transition amplitude from one meson state to any many mesons state must be real at leading order of N_c^{-1} expansion,

$$\text{Im}(\mathcal{T}_{\beta,\alpha}^{(0)})_{(k-1)} = 0, \quad (8.7)$$

where the superscript (0) denotes leading order.

In the following we shall explicitly examine eq. (8.1) in forward scattering of ρ -meson up to two-loop level of mesons. The examination of other processes can be performed similarly. For the case $\alpha = \beta = \rho$, $< \gamma | = < \pi\pi |$ is dominant. Then for forward scattering of ρ -meson, eq. (8.1) becomes

$$\frac{2}{(2\pi)^4} \text{Im} \mathcal{T}_{\rho,\rho} = \Gamma(\rho \rightarrow \pi\pi) = \frac{4}{3\pi} \left[\frac{\alpha(m_\rho^2) f_{111}^{(c)}(m_\rho^2, m_\pi^2, m_\pi^2)}{g F_\pi^2} \right]^2 m_\rho \left[1 - 4 \frac{m_\pi^2}{m_\rho^2} \right]^{3/2}, \quad (8.8)$$

and the expansions of $\mathcal{T}_{\rho,\rho}$ and $f_{111}^{(c)}$ are

$$\mathcal{T}_{\rho,\rho} = \sum_{n=0}^{\infty} (\mathcal{T}_{\rho,\rho})_{2n}, \quad f_{111}^{(c)} = \sum_{n=0}^{\infty} (f_{111}^{(c)})_{2n+1}. \quad (8.9)$$

At the leading order, $\text{Im}(\mathcal{T}_{\rho,\rho})_0 = 0$ is obviously satisfied. To obtain the imaginary part of $(\mathcal{T}_{\rho,\rho})_2$, we need to calculate meson loop correction in fig. 5. The calculations are similar to those in Sect. VI, and the result is

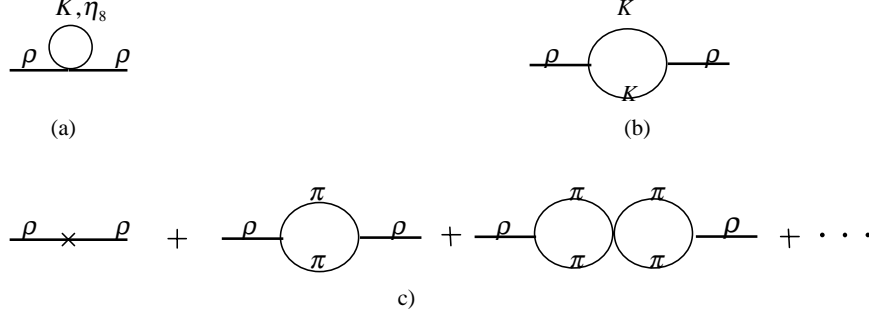


FIG. 5. One-loop diagrams correcting to ρ propagator. a) Tadpole diagram of K or η_8 . b) Two-point diagram of K . c) Chain-like approximation of pion.

$$\mathcal{L}_{\rho\rho}^{1-loop} = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot x} [\alpha(p^2)]^2 \frac{64}{g^2} \left\{ \frac{[f_{111}^{(0)}(p^2)]^2}{F_\pi^4} \frac{\Sigma_\pi^{(0)}(p^2)}{1 + \Sigma_\pi^{(0)}(p^2)} - \frac{[f_{113}^{(0)}(p^2)]^2}{F_K^4 p^2} m_K^2 \Sigma_K^{(0)}(p^2) \right\} (g_{\mu\nu} p^2 - p_\mu p_\nu) \rho_i^\mu(p) \rho_i^\nu(x), \quad (8.10)$$

where

$$\begin{aligned} \Sigma_\pi^{(0)}(p^2) &= \frac{1}{8\pi^2} \left\{ \frac{\lambda}{6} + \int_0^1 dx \cdot x(1-x) \ln \frac{x(1-x)p^2}{m_K^2} + \frac{i}{6} \text{Arg}(-1) \theta(p^2 - 4m_\pi^2) \left(1 - \frac{4m_\pi^2}{p^2} \right)^{3/2} \right\}, \\ \Sigma_K^{(0)}(p^2) &= \frac{1}{16\pi^2} \left\{ \lambda \left(1 - \frac{p^2}{6m_K^2} \right) + \int_0^1 dx \left(1 - \frac{x(1-x)p^2}{m_K^2} \right) \ln \left(1 - \frac{x(1-x)p^2}{m_K^2} \right) \right\}. \end{aligned} \quad (8.11)$$

Taking $\text{Arg}(-1) = -\pi$, we obtain

$$\frac{2}{(2\pi)^4} \text{Im}(\mathcal{T}_{\rho,\rho})_2 = \frac{4}{3\pi} \left[\frac{\alpha(m_\rho^2) f_{111}^{(0)}(m_\rho^2, m_\pi^2, m_\pi^2)}{g F_\pi^2} \right]^2 m_\rho \left[1 - 4 \frac{m_\pi^2}{m_\rho^2} \right]^{3/2}. \quad (8.12)$$

Noting that $(f_{111}^{(c)})_1 = f_{111}^{(0)}$, we can see the eq. (8.8) is satisfied at the next to leading order.

If the two-loop diagrams and three-loop diagrams are further considered, we can prove the eq. (8.8) is satisfied up to $O(N_c^{-3})$. The details of calculation are omitted here.

IX. DETERMINATION OF Λ_{CSSB}

In the Section above, the unitarity of the effective meson theory reduced from ChQM in terms of resummation calculations has been investigated. By using optical theorem and large- N_c expansion, a theorem has been proved in framework of ChQM and at the leading order of N_c^{-1} expansion that the on-shell transition amplitude $\mathcal{T}_{\beta,\alpha}^{(0)}$ (superscript (0) denotes leading order of N_c^{-1} expansion) must be real for transition from one meson state α to any many mesons state β , i.e.,

$$\text{Im}(\mathcal{T}_{\beta,\alpha}^{(0)}) = 0, \quad (9.1)$$

which, as we shall see, represents a nontrivial restriction on the theory by the unitarity of S -matrix.

It is easy to see that, for $\alpha = \beta = V$ or Φ , we have

$$\text{Im}\mathcal{T}_{V,V}^{(0)} = 0, \quad \text{Im}\mathcal{T}_{\Phi,\Phi}^{(0)} = 0. \quad (9.2)$$

which satisfies the requirement of eq. (9.1). Next, we examine $\text{Im}\mathcal{T}_{\Phi\Phi,V}^{(0)}$. To calculate transition amplitude from one vector meson state ($\alpha = V$) to two pseudoscalar mesons state ($\beta = \{\Phi, \Phi\}$), we need Feynman rules (see fig. 6) for vertex $V - \Phi\Phi$, which can be read from action (5.10) of the effective theory, and this is at the leading order of N_c^{-1} expansion.

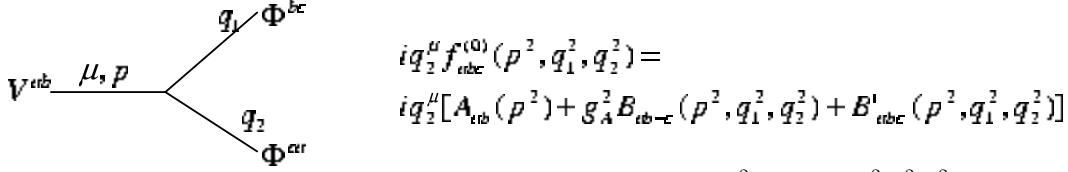


FIG. 6. Feynman rule of vertex $V - \Phi\Phi$, where $A_{ab}(p^2)$, $B_{ab-c}(p^2, q_1^2, q_2^2)$ and $B'_{abc}(p^2, q_1^2, q_2^2)$ are given by eqs. (5.2), (5.5) and (5.7) respectively.

We like to address that, being compared with ordinary quantum field theories such as QED, QCD or ϕ^4 -theory, the vertex function of our effective meson theory is much more complicated. Especially, there are two objects in it (see eq. (5.2), (5.5) and (5.7)),

$$\ln \frac{D_1^{ab}}{m^2} = \ln \frac{M_a^2(1-x) + M_b^2x}{m^2} + \ln \left(1 - \frac{p^2x(1-x)}{M_a^2(1-x) + M_b^2x} \right),$$

$$\ln \frac{D_2^{abc}}{m^2} = \ln \frac{M_a^2x(1-y) + M_b^2(1-x) + M_c^2xy}{m^2} + \ln \left(1 - \frac{p^2x(1-x)(1-y) + q_1^2xy(1-x) + q_2^2x^2y(1-y)}{M_a^2x(1-y) + M_b^2(1-x) + M_c^2xy} \right).$$

There is no such kind of logarithm terms in the Feynman rules of ordinary fundamental quantum field theories, even in other kinds of effective theories. This is a remarkably new feature. These two logarithm terms result directly from resummation calculation and they reflect non-perturbative effects, as should be emphasized. There is no such kind of structure in any $O(p^4)$ -, or $O(p^6)$ -effective meson theories.

Using the Feynman rule Fig.(6), we get transition amplitude for $V \rightarrow 2\Phi$ at the leading order of N_c^{-1} expansion as follows

$$\mathcal{T}_{\Phi\Phi,V}^{(0)} \equiv \langle \Phi^{bc}(q_1)\Phi^{ca}(q_2) | \mathcal{T}^{(0)} | V^{ab}(p, \lambda) \rangle = (2\pi)^4 \delta^4(p - q_1 - q_2) q_2^\mu \epsilon_\mu^\lambda f_{abc}^{(0)}(p^2, q_1^2, q_2^2). \quad (9.3)$$

where ϵ_μ^λ is the polarization vector of the vector meson $V^{ab}(p, \lambda)$. Then, the imaginary part of $\mathcal{T}_{\Phi\Phi,V}^{(0)}$ reads

$$\begin{aligned} \text{Im}\mathcal{T}_{\Phi\Phi,V}^{(0)} &= \int_0^1 dx f(x, p) \text{Im} \left[\ln \left(1 - \frac{p^2x(1-x)}{M_a^2(1-x) + M_b^2x} \right) \right] \\ &+ \int_0^1 dx \int_0^1 dy g(x, y, p, q_1, q_2) \text{Im} \left[\ln \left(1 - \frac{p^2x(1-x)(1-y) + q_1^2xy(1-x) + q_2^2x^2y(1-y)}{M_a^2x(1-y) + M_b^2(1-x) + M_c^2xy} \right) \right] \end{aligned} \quad (9.4)$$

where $f(x, p)$ and $g(x, y, p, q_1, q_2)$ are definite real functions. From theorem (9.1), $\text{Im}\mathcal{T}_{\Phi\Phi,V}^{(0)}$ has been required to be 0, otherwise, the unitarity of the theory will be broken down. Consequently, we have

$$D_1^{ab}(p^2) \equiv M_a^2(1-x) + M_b^2x - p^2x(1-x) > 0, \quad (0 \leq x \leq 1),$$

and

$$D_2^{abc} \equiv M_a^2x(1-y) + M_b^2(1-x) + M_c^2xy - p^2x(1-x)(1-y) - q_1^2xy(1-x) - q_2^2x^2y(1-y) > 0, \quad (0 \leq x, y \leq 1).$$

This will lead to a restriction on the range of p^2 . The former inequality will hold in domain $0 \leq x \leq 1$ if

$$M_{V^{ab}} = \sqrt{p^2} \leq M_a + M_b.$$

As to the latter, the right side of it has no stationary point in $x - y$ plane, therefore this inequality holding in the square domain is equivalent to it holding at boundary of the square, which gives

$$\begin{cases} M_{V^{ab}} = \sqrt{p^2} \leq M_a + M_b, \\ M_{\Phi^{ab}} = \sqrt{q^2} \leq M_a + M_b. \end{cases}$$

Because $M_{\Phi^{ab}} < M_{V^{ab}}$, we see that the second condition is satisfied if the first one does. Therefore we conclude that the necessary condition for the effective theory to be unitary is

$$M_{V^{ab}} = \sqrt{p^2} \leq 2m + m_a + m_b \equiv \Lambda^{ab}. \quad (9.5)$$

where $M_a = m + m_a$ ($a = u, d, s$) have been used.

Specifically, setting $(ab) = (ud)$ (i.e., $V^{ab} = \rho$, $M_{V^{ab}} = m_\rho$) and $m_u = m_d = 0$ in eq.(9.5), we see that, in ChQM with ρ meson resonances, the unitarity of S -matrix requires mass of constituent quark $m \geq m_\rho/2$ [19]. In fact, this requirement is ensured in ChQM by $m \simeq 460\text{MeV}$ obtained by fitting the low energy limit of the model (see Appendix A). For $V = K^*(892)$ and $\phi(1020)$ cases, $m_s \simeq 170\text{MeV}$ have to be taken into account, and the corresponding unitarity conditions are $m \geq (m_{K^*} - m_s)/2$ and $m \geq (m_\phi - 2m_s)/2$ respectively. They are also satisfied by taking $m \simeq 460\text{MeV}$.

In the above discussions, we have actually revealed an important fact that $\Lambda^{ab} \equiv 2m + m_a + m_b$ is a critical energy scale in the effective meson theory of ChQM. As $\sqrt{p^2}$ is below Λ^{ab} , the S -matrices yielded from the Feynman rules of the meson theory are unitary, while as $\sqrt{p^2}$ is above this scale, the unitarity of the meson theory will be broken down. This fact indicates that the well-defined effective quantum field theory describing the meson physics in the framework of ChQM exists only as the typical energies are below Λ^{ab} . When energy is above Λ^{ab} , the effective meson-Lagrangian description of the dynamics is illegal in principle because the unitarity fails. This is precisely a critical phenomenon, or quantum phase transition in quantum field theory, which is caused by quantum fluctuations in the system [8]. Recalling the meaning of the scale Λ_{CSSB} of chiral symmetry spontaneously breaking in QCD, we can see that Λ^{ab} play the same role as Λ_{CSSB} . Then, in the framework of ChQM we have

$$\Lambda_{\text{CSSB}} = \Lambda^{ab} \equiv 2m + m_a + m_b. \quad (9.6)$$

It is essential here that Λ_{CSSB} is flavor-dependent. Numerically, for ud -flavor system (e.g., $\pi - \rho - \omega$ physics),

$$\Lambda_{\text{CSSB}}(ud) \simeq 2m = 920\text{MeV}, \quad (9.7)$$

for $u(d)s$ -flavor system (e.g., $K - K^*$ physics),

$$\Lambda_{\text{CSSB}}(u(d)s) \simeq 2m + m_s = 1090\text{MeV}, \quad (9.8)$$

for $\bar{s}s$ case (e.g., ϕ -physics),

$$\Lambda_{\text{CSSB}}(ss) \simeq 2(m + m_s) = 1260\text{MeV}. \quad (9.9)$$

Since $m_\rho < \Lambda_{\text{CSSB}}(ud)$, $m_{K^*} < \Lambda_{\text{CSSB}}(u(d)s)$ and $m_\phi < \Lambda_{\text{CSSB}}(ss)$, the effective meson field theory derived by resummation derivation in ChQM in this paper is unitary. And the low energy expansions of p are legitimate and convergent due to $p^2/\Lambda_{\text{CSSB}}^2 < 1$. It means that all light flavor vector meson resonances can be included in ChQM model consistently. It is remarkable that the quantum phase transitions in ChQM can be explored successfully in resummation derivation method, and the corresponding critical scales are determined analytically.

In ref. [3], Λ_{CSSB} has been estimated by comparing $\mathcal{O}(p^2)$ contributions to $\mathcal{O}(p^4)$'s for $\pi - \pi$ scattering process, and it has been shown that Λ_{CSSB} is around $2\pi F_\pi \sim 1.2\text{GeV}$. However, the quantum phase transitions in ChQM were not explored in ref. [3], and the existence of Λ_{CSSB} is a prior hypothesis there. Therefore the studies on determination of Λ_{CSSB} in this present paper is significantly different from ones in [3], even though the numerical values of Λ_{CSSB} both in [3] and in this paper are closing.

X. SUMMARY AND DISCUSSION

In this paper, starting from the chiral constitute quark model with the lowest vector meson resonances we have achieved resummation studies on the processes of $\rho \rightarrow \pi\pi$, $\rho^0 \rightarrow e^+e^-$, $K^* \rightarrow K\pi$, $\phi \rightarrow K^+K^-$ and $\phi \rightarrow K_L^0 K_S^0$, and the results are in good agreement with the data. The error is less than 17%. In our calculations, all chiral expansion

(or p -expansion) terms have been included, and analytic expressions for decay widths of these processes are obtained. Distinguishing from the ChQM-effective Lagrangian derivations existed in the literature [12,14,15], we have calculated not only the quark loops, but also the Goldstone boson loops at one-loop level. Thus, besides the contributions of leading order of $1/N_c$ -expansion, ones due to the next to the leading order of $1/N_c$ have also been taken into account. The logarithmic divergence due to quark loops and the quadratic divergence due to meson loops have been absorbed by the parameters g and λ respectively, while the values of g and λ are determined by the KSUF sum rule and by the Zweig rule forbidden to $\phi \rightarrow \pi\pi$ respectively. The fact that this QCD-inspired parametrization leads to the reasonable results indicates that both the model employed by us and the derivations in this paper are legitimate and consistent.

In the Introduction, we have addressed the resummation studies are necessary for the vector meson resonance physics because it is related to the question whether the chiral dynamical description is legitimate or not. The success of such studies achieved in this paper provides a meaningful evidence that the chiral Lagrangian method works at this energy region. This is one of the main conclusions of this paper. It should be addressed again that this conclusion can not be reached from any so called $O(p^4)$ - or $O(p^6)$ -studies on any chiral Lagrangian theories or models with vector meson resonances. Actually, any predictions coming from a chiral Lagrangian with only few terms in chiral expansion in this energy region are not reliable. In the TABLE I, we have shown that the contributions coming from high-order terms of p -expansion are important. It implies that the feasibility to evaluate high order contribution of momentum expansion has to be ensured for any practically working effective meson theories with vector meson resonances. In addition we have also point out in the Introduction that the contributions of the next to leading order in $1/N_c$ -expansion have also to be considered, otherwise the loop calculations will be incomplete. Our calculations show that the situation is as expected indeed. The next to leading order contributions in the expansion of $1/N_c$ makes the predictions more closing to the data significantly (see TABLE I).

The unitarity of the effective meson theory including all p -expansion terms deduced from ChQM has been investigated by means of the optic theorem and $1/N_c$ expansion argument in QCD. It has been found that the necessary condition for the unitarity of the theory is the momentum p of vector meson V^{ab} satisfies $\sqrt{p^2} < \Lambda^{ab} \equiv 2m + m_a + m_b$, otherwise, as $\sqrt{p^2}$ is above Λ^{ab} , the unitarity will be broken down. Then, we conclude that the chiral symmetry spontaneously broken scale $\Lambda_{\text{CSSB}} = \Lambda^{ab}$. It is clear that this represents an explicit study on a quantum critical phenomenon. Actually, we are working in order phase (or meson field phase), i.e., the order parameter is constitute quark mass $m \neq 0$, dynamical quantum field freedoms are meson fields and the quantum fluctuation parameter is $\mathcal{G} = p^2/(\Lambda^{ab})^2$. And then we revealed that the critic point is $\mathcal{G}_c = 1$. Obviously, our method to study CSSB is significantly different from ones in [5] [6] where the gap equation (or Schwinger-Dyson equation) is used for understanding CSSB.

An important feature of this result is that Λ_{CSSB} in ChQM is flavor-dependent, i.e., $\Lambda_{\text{CSSB}}(ud) \neq \Lambda_{\text{CSSB}}((ud)s) \neq \Lambda_{\text{CSSB}}(ss)$. This result should be interesting and meaningful in physics because the critical energy scale is one of quantities to characterize the intrinsic properties of the physical system, and hence it should be dependent of the contents on the system. Furthermore, that $m_\rho < \Lambda_{\text{CSSB}}(ud)$, $m_{K^*} < \Lambda_{\text{CSSB}}((ud)s)$ and $m_\phi < \Lambda_{\text{CSSB}}(ss)$ indicates that the studies on the vector meson decays presented in this paper in the chiral effective meson field theory of ChQM are legitimate and self-consistent.

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APPENDIX A: LOW ENERGY LIMIT

It is well known that, at very low energy, ChPT is a rigorous consequence of the symmetry pattern of QCD and its spontaneous breaking. Therefore the low energy limit of any models concerning meson resonances must match with ChPT. The low energy limit of ChQM model can be obtained via integrating over vector meson resonances. It means that, at very low energy, the dynamics of vector mesons are replaced by pseudoscalar meson fields. Since there are no interaction of vector mesons at $O(p^2)$, at very low energy, the equation of motion $\delta\mathcal{L}/\delta V_\mu = 0$ yields classical solution for vector mesons

$$V_\mu = \frac{1}{m_v^2} \times \text{terms of } O(p^3), \quad (1.1)$$

where p is momentum of pseudoscalar at very low energy. Therefore, in effective action S_n , the terms involving vector meson resonances are $O(p^6)$ at very low energy and do not contribute to $O(p^4)$ low energy coupling constants, L_i ($i = 1, 2, \dots, 10$). The low energy coupling constants L_i ($i \neq 7$) yielded by ChQM model can be directly obtained from effective actions resulted from quark loop,

$$\begin{aligned} L_1 &= \frac{1}{2}L_2 = \frac{1}{128\pi^2}, & L_3 &= -\frac{3}{64\pi^2} + \frac{1}{64\pi^2}g_A^4, \\ L_4 &= L_6 = 0, & L_5 &= \frac{3m}{32\pi^2 B_0}g_A^2, \\ L_8 &= \frac{F_0^2}{128B_0m}(3 - \kappa^2) + \frac{3m}{64\pi^2 B_0}\left(\frac{m}{B_0} - \kappa g_A - \frac{g_A^2}{2} - \frac{B_0}{6m}g_A^2\right) + \frac{L_5}{2}, \\ L_9 &= \frac{1}{16\pi^2}, & L_{10} &= -\frac{1}{16\pi^2} + \frac{1}{32\pi^2}g_A^2. \end{aligned} \quad (1.2)$$

In fact, the above expression on L_i have been obtained in some previous refs. [3,13,26] (besides of L_8).

The constants L_7 has been known to get dominant contribution from η_0 [4] and this contribution is suppressed by $1/N_c$. If we ignore the $\eta - \eta'$ mixing, we have

$$L_7 = -\frac{f_\pi^2}{128m_{\eta'}^2}. \quad (1.3)$$

Thus six free parameters, g (fitted by KSRF sum rule), g_A (fitted by $n \rightarrow pe^- \bar{\nu}_e$ decay), B_0 , κ , m and $m_{\eta'}$ determine all ten low energy coupling constants of ChPT. It reflects the dynamics constrains between those low energy coupling constants. Moreover, if we take $m_u + m_d \simeq 11\text{MeV}$, we can obtain $B_0 = \frac{m_\pi^2}{m_u + m_d} \simeq 1.8\text{GeV}$. Then experimental values of L_5 constrains constituent quark mass $m \simeq 460\text{MeV}$. Setting $F_0 = F_\pi$ and using L_8 as input, we find that $\kappa \simeq 0.5$. The numerical results for these low energy constants are in table II.

	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	L_9	L_{10}
ChPT	0.7 ± 0.3	1.3 ± 0.7	-4.4 ± 2.5	-0.3 ± 0.5	1.4 ± 0.5	-0.2 ± 0.3	-0.4 ± 0.15	0.9 ± 0.3	6.9 ± 0.7	-5.2 ± 0.3
ChQM	0.79	1.58	-4.25	0	$1.4^a)$	0	$(-0.4 \pm 0.1)^b)$	$0.9^a)$	6.33	-4.55

TABLE II. L_i in units of 10^{-3} , $\mu = m_\rho$. a)input. b)contribution from gluon anomaly.

APPENDIX B: CANCELLATION OF QUADRATIC DIVERGENCE OF MESON LOOPS

From calculations in Sect. VI, we can find that only quadratic divergence appears in one-loop contribution of pseudoscalar mesons. Since the present model is a non-renormalizable effective theory, the divergences have to be factorized, i.e., the parameter λ has to be determined phenomenologically.

The on-shell decay $\phi \rightarrow \pi\pi$ is forbidden by G parity conservation and Zweig rule. Experiment also show that branching ratios of this decay is very small, $B(\phi \rightarrow \pi\pi) = (8 \pm 5) \times 10^{-5}$. Theoretically, this decay can occur through photon-exchange or K -loop (fig.5). The latter two diagrams yield non-zero imaginary part of decay amplitude. Thus the real part yielded by the latter two diagrams must be very small. We can determine λ due to this requirement. From the calculation in the above two subsection, we see that result yielded by the latter two diagrams is proportional to a factor

$$\lambda\left(\frac{p^2}{6} - m_\kappa^2\right) - \int_0^1 dx \cdot [m_\kappa^2 - x(1-x)p^2] \ln\left(1 - \frac{x(1-x)p^2}{m_\kappa^2}\right). \quad (2.1)$$

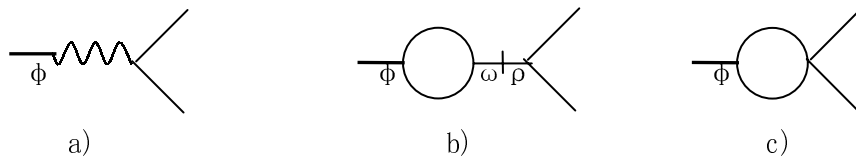


FIG. 7. Some diagrams for $\phi \rightarrow \pi\pi$ decay. The one-loop in figure b) and c) is K -loop.

Then Zweig rule requires

$$\left\{ \lambda\left(\frac{p^2}{6} - m_\kappa^2\right) - \text{Re} \int_0^1 dx \cdot [m_\kappa^2 - x(1-x)p^2] \ln\left(1 - \frac{x(1-x)p^2}{m_\kappa^2}\right) \right\} \Big|_{p^2=m_\phi^2} \simeq 0. \quad (2.2)$$

Form the above equation, we obtain $\lambda \simeq 0.54$.